S.131/ 4

4. a) $f'(x) = 2 \cdot (1 + 3x^2) \cdot 2 \cdot 3x = 12x(1 + 3x^2);$

$f'(2) = 12 \cdot 2\,(1 + 3 \cdot 4) = 24 \cdot 13 = 312$

b) $f'(x) = 2 \cdot (1 - x^4) \cdot (-4x^3) = -8x^3 \cdot (1 - x^4);$

$f'(1) = -8 \cdot (1 - 1) = 0$

c) $f'(x) = 2 \cdot (1 + 2x + x^2) \cdot (2 + 2x) = 4 \cdot (1 + x)^2 \cdot (1 + x) = 4 \cdot (1 + x)^3;$

$f'(-1) = 4 \cdot (1 + (-1))^3 = 0$

d) $f'(x) = 2 \cdot (1 - 5x) \cdot (-5) = -10 \cdot (1 - 5x);$

$f'(0) = -10 \cdot (1 - 0) = -10$

e) $f'(x) = 2 \cdot (x^5 + 1) \cdot 5x = 10x^4(1 + x^5);$

$f'(-1) = 10 \cdot (1 + (-1)) = 0$

4)

f) $f'(x) = 2 \cdot [x - (3x)^2] \cdot (1 - 2 \cdot 3x \cdot 3) = (2 - 36x) \cdot (x - 9x^2);$

$f'(2) = (2 - 72) \cdot (2 - 9 \cdot 4) = -70 \cdot (-34) = 2\,380$

g) $f'(x) = 2 \cdot \left(2x + \frac{1}{4x}\right) \cdot \left(2 - \frac{4}{(4x)^2}\right) = \left(4x + \frac{1}{2x}\right) \cdot \left(2 - \frac{1}{4x^2}\right);$

$f'(0{,}5) = \left(4 \cdot 0{,}5 + \frac{1}{2 \cdot 0{,}5}\right) \cdot \left(2 - \frac{1}{4 \cdot (0{,}5)^2}\right) = 3 \cdot 1 = 3$

h) $f'(x) = -2 \cdot (1 + 4x)^{-3} \cdot 4 = -\frac{8}{(1 + 4x)^3};$

$f'(0{,}5) = -\frac{8}{(1 + 4 \cdot 0{,}5)^3} = -\frac{8}{27};$

i) $f'(x) = 2 \cdot \left(\frac{2x + 1}{1 - 4x}\right) \cdot \frac{(1 - 4x) \cdot 2 - (2x + 1) \cdot (-4)}{(1 - 4x)^2} = \frac{4x + 2}{1 - 4x} \cdot \frac{2 - 8x + 8x + 4}{(1 - 4x)^2};$

$= \frac{(4x + 2) \cdot 6}{(1 - 4x)^3} = \frac{24x + 12}{(1 - 4x)^3}$

$f'(-0{,}5) = \frac{24 \cdot (-0{,}5) + 12}{(1 - 4 \cdot (-0{,}5))^3} = 0$

j) $f'(x) = -4 \cdot (9 - x^2)^2 + (2 - 4x) \cdot 2 \cdot (9 - x^2) \cdot (-2x)$

$= (9 - x^2) \cdot [-36 + 4x^2 - 8x + 16x^2]$

$= (9 - x^2) \cdot (20x^2 - 8x - 36);$

$f'(3) = (9 - 9) \cdot (20 \cdot 9 - 8 \cdot 9 - 36) = 0$

k) $f'(x) = 4 \cdot (1 + x^2)^3 \cdot 2x = 8x(1 + x^2)^3;$

$f'(2) = 8 \cdot 2\,(1 + 4)^3 = 16 \cdot 125 = 2\,000$

l) $f'(x) = n \cdot (4 - x)^{n-1} \cdot (-1) = -n \cdot (4 - x)^{n-1};$

$f'(4) = -n \cdot (4 - 4)^{n-1} = 0$

m) $f'(x) = \frac{(x^2 + 1)^3 \cdot 1 - x \cdot 3 \cdot (x^2 + 1)^2 \cdot 2x}{(x^2 + 1)^6} = \frac{x^2 + 1 - 6x^2}{(x^2 + 1)^4} = \frac{1 - 5x^2}{(x^2 + 1)^4};$

$f'(0) = \frac{1 - 5 \cdot 0}{(0 + 1)^4} = 1$

n) $f'(x) = 2 \cdot \left(\frac{1 - x}{1 + x^2}\right) \cdot \frac{(1 + x^2) \cdot (-1) - (1 - x) \cdot 2x}{(1 + x^2)^2} = \frac{2 - 2x}{1 + x^2} \cdot \frac{-1 + x^2 - 2x}{(1 + x^2)^2}$

$= \frac{(2 - 2x) \cdot (x^2 - 2x - 1)}{(1 + x^2)^3};$

$f'(0) = \frac{(2 - 0) \cdot (-1)}{(1 + 0)^3} = -2$

o) $f'(x) = -\frac{4 \cdot 2 \cdot (4 + x^2) \cdot 2x}{(4 + x^2)^4} = -\frac{16x}{(4 + x^2)^3};$

$f'(-2) = \frac{32}{(4 + 4)^3} = \frac{32}{512} = \frac{1}{16}$

p) $f'(x) = 2 \cdot \left(\frac{1 + x^2}{1 - x}\right) \cdot \frac{(1 - x) \cdot 2x - (1 + x^2) \cdot (-1)}{(1 - x)^2} = \frac{2 + 2x^2}{1 - x} \cdot \frac{2x - 2x^2 + 1 + x^2}{(1 - x)^2}$

$= \frac{(2 + 2x^2) \cdot (-x^2 + 2x + 1)}{(1 - x)^3};$

$f'(-1) = \frac{(2 + 2) \cdot (-1 - 2 + 1)}{(1 + 1)^3} = \frac{-8}{8} = -1$

S. 134/ 1

1. a) $y' = -\sin(x - 3)$

b) $y' = \cos(x^2) \cdot 2x = 2x \cdot \cos(x^2)$

c) $y' = \cos x \cdot \cos x + \sin x \cdot (-\sin x) = (\cos x)^2 - (\sin x)^2$

d) $y' = 2 \cdot \sin x \cdot \cos x$

e) $y' = 2 \cdot \cos x \cdot (-\sin x) = -2\sin x \cdot \cos x$

f) $y' = -\sin(ax + b) \cdot a = -a \cdot \sin(ax + b)$

g) $y' = 2x \cdot \sin x + x^2 \cdot \cos x$

h) $y = (\cos x)^2 \cdot (\cos x)^2 = (\cos x)^4$;
$y' = 4(\cos x)^3 \cdot (-\sin x) = -4 \sin x \cdot (\cos x)^3$

i) $y = 1; y' = 0$

j) $y = x^3 + x^2 \cdot (-1) + x(-1) + \frac{1}{2}\sqrt{2} = x^3 - x^2 - x + \frac{1}{2}\sqrt{2}$;
$y' = 3x^2 - 2x - 1$

n) $y' = \frac{\sin x - x \cdot \cos x}{(\sin x)^2}$

o) $y' = \left(-\sin\frac{1}{x}\right) \cdot \left(-\frac{1}{x^2}\right) = \frac{1}{x^2} \cdot \sin\frac{1}{x}$

p) $y' = \cos\left(\frac{\pi}{3} \cdot x\right) \cdot \frac{\pi}{3} = \frac{\pi}{3} \cdot \cos\left(\frac{\pi}{3} \cdot x\right)$

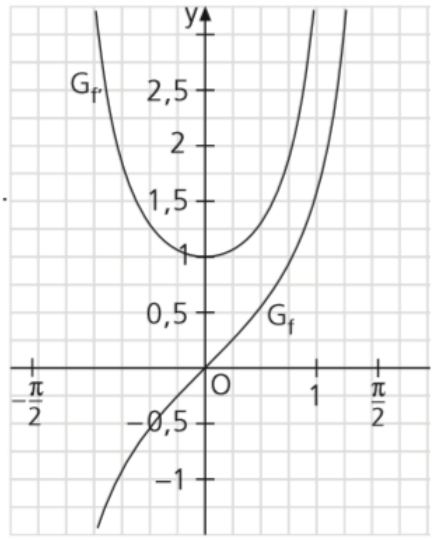
S. 135 / 3c

c) $f(x) = \frac{\sin x}{\cos x}$; $f'(x) = \frac{1}{(\cos x)^2}$; $D_f =]-\frac{\pi}{2}; \frac{\pi}{2}[$

(1) $\frac{1}{(\cos x)^2} = 0$; G_f hat keinen Extrempunkt.

(2) $\frac{1}{(\cos x)^2} = 1$; $\cos x = \pm 1$; $x = 0 \in D_f$

Im Ursprung hat G_f eine Tangente mit der Steigung 1.



S.135 / 4 c

4. a) $F(x) = -\frac{1}{2}\cos(2x)$

Probe: $F'(x) = -\frac{1}{2}(-\sin 2x) \cdot 2 = \sin(2x) = f(x)$ ✓

b) $F(x) = \pi \cdot \sin\frac{x}{\pi}$

Probe: $F'(x) = \pi \cdot \left(\cos\frac{x}{\pi}\right) \cdot \frac{1}{\pi} = \cos\frac{x}{\pi} = f(x)$ ✓

c) $F(x) = \frac{1}{2}x^2 + \sin x$

Probe: $F'(x) = x + \cos x = f(x)$ ✓

d) $F(x) = -\frac{40}{\pi} \cdot \cos\left(\frac{\pi}{4} \cdot x\right)$

Probe: $F'(x) = -\frac{40}{\pi} \cdot \left(-\sin\left(\frac{\pi}{4}x\right)\right) \cdot \frac{\pi}{4} = 10 \cdot \sin\left(\frac{\pi}{4}x\right) = f(x)$ ✓

S. 140/1

1. a) $f'(x) = \frac{1}{4} \cdot x^{\frac{1}{4}-1} = \frac{1}{4} \cdot x^{-\frac{3}{4}} = \frac{1}{4} \cdot \frac{1}{\sqrt[4]{x^3}}$

b) $f'(x) = \frac{3}{4} \cdot x^{\frac{3}{4}-1} = \frac{3}{4} \cdot x^{-\frac{1}{4}} = \frac{3}{4} \cdot \frac{1}{\sqrt[4]{x}}$

c) $f(x) = \sqrt{3x^2} = \sqrt{3} \cdot \left(\sqrt{x}\right)^2 = \sqrt{3} \cdot x$ (da $x > 0$)

$f'(x) = \sqrt{3}$

d) $f(x) = \sqrt[3]{x^6} = \left(\sqrt[3]{x}\right)^6 = \sqrt{x} = x^{\frac{1}{2}}$ (da $x > 0$)

$f'(x) = \frac{1}{2} \cdot x^{\frac{1}{2}-1} = \frac{1}{2} \cdot x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$

e) $f(x) = \left(\sqrt{2x}\right)^{-1} = (2x)^{-\frac{1}{2}}$

$f'(x) = -\frac{1}{2} \cdot (2x)^{-\frac{1}{2}-1} \cdot 2 = -(2x)^{-\frac{3}{2}} = -\frac{1}{\sqrt{(2x)^3}}$

f) $f(x) = x^{\frac{1}{2}} \cdot \sin x$

$f'(x) = \frac{1}{2} \cdot x^{\frac{1}{2}-1} \cdot \sin x + x^{\frac{1}{2}} \cdot \cos x = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} \cdot \sin x + \sqrt{x} \cdot \cos x$

g) $f(x) = (4x^2 + 1)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2} \cdot (4x^2 + 1)^{-\frac{1}{2}} \cdot 8x = \frac{4x}{\sqrt{4x^2 + 1}}$