Delta 11 Lösungen S. 144 / 3b, c; 5; 6

b) (1) Produktregel: $f(x) = (1-x)\cdot(4-2x)^2$

$f'(x) = (-1)\cdot(4-2x)^2 + (1-x)\cdot 2\cdot(4-2x)\cdot(-2) = -(4-2x)^2 + (-4+4x)\cdot(4-2x) =$

$= -16 + 16x - 4x^2 - 16 + 8x + 16x - 8x^2 = -12x^2 + 40x - 32$

(2) Ausmultipliziert:

$f(x) = (1-x)\cdot(4-2x)^2 = (1-x)\cdot(16-16x+4x^2) =$

$= 16 - 16x + 4x^2 - 16x + 16x^2 - 4x^3 = -4x^3 + 20x^2 - 32x + 16$

$f'(x) = -12x^2 + 40x - 32$

$D_{f\,max} = \mathbb{R}$

$f'(x) = -12x^2 + 40x - 32 = 0$

$x_{1,2} = \frac{-40 \pm \sqrt{1600 - 4\cdot(-12)\cdot(-32)}}{2\cdot(-12)} = \frac{-40 \pm 8}{-24}$

$x_1 = \frac{4}{3};\ y_1 = f\left(\frac{4}{3}\right) = -\frac{1}{3}\cdot\frac{16}{9} = -\frac{16}{27};\ x_2 = 2;\ y_2 = f(2) = 0$

Monotonietabelle:

x	$x < \frac{4}{3}$	$x = \frac{4}{3}$	$\frac{4}{3} < x < 2$	$x = 2$	$x > 2$
f´(x)	f´(x) < 0	f´(x) = 0	f´(x) > 0	f´(x) = 0	f´(x) < 0
G_f	smf	Tiefpunkt $\left(\frac{4}{3}/-\frac{16}{27}\right)$	sms	Hochpunkt (2/0)	smf

c) (1) Kettenregel: $f(x) = \left(1 - \frac{1}{x}\right)^2 = (1 - x^{-1})^2$

$f'(x) = 2\cdot(1-x^{-1})\cdot x^{-2} = 2\cdot\frac{1}{x^2}\cdot\left(1-\frac{1}{x}\right) = \frac{2}{x^2} - \frac{2}{x^3} = \frac{2x-2}{x^3}$

(2) Produktregel: $f(x) = (1-x^{-1})\cdot(1-x^{-1})$

$f'(x) = x^{-2}\cdot(1-x^{-1}) + (1-x^{-1})\cdot x^{-2} = \frac{1}{x^2}\cdot\left(1-\frac{1}{x}\right) + \frac{1}{x^2}\cdot\left(1-\frac{1}{x}\right) =$

$= \frac{2}{x^2}\cdot\left(1-\frac{1}{x}\right) = \frac{2}{x^2} - \frac{2}{x^3} = \frac{2x-2}{x^3}$

(3) Ausmultiplizieren: $f(x) = \left(1-\frac{1}{x}\right)^2 = 1 - \frac{2}{x} + \frac{1}{x^2} = 1 - 2\cdot x^{-1} + x^2$

$f'(x) = 2\cdot x^{-2} - 2\cdot x^{-3} = \frac{2}{x^2} - \frac{2}{x^3} = \frac{2x-2}{x^3}$

$D_{f\,max} = \mathbb{R}\setminus\{0\}$

$f'(x) = \frac{2x-2}{x^3} = 0$

$2x - 2 = 0$

$x = 1;\quad y = f(1) = 0$

Monotonietabelle:

x	x < 1	x = 1	x > 1
f´(x)	f´(x) < 0	f´(x) = 0	f´(x) > 0
G_f	smf	Tiefpunkt (1/0)	sms

5. $F(x) = 1 + x + \sin x \cdot \cos x;$

$F'(x) = 1 + \cos x \cdot \cos x + \sin x \cdot (-\sin x) =$

$= 1 + (\cos x)^2 - (\sin x)^2 =$

$= [1 - (\sin x)^2] + (\cos x)^2 =$

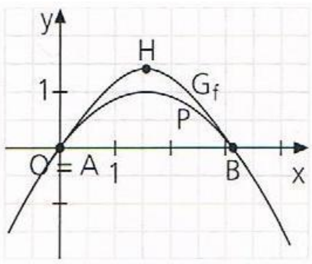
$= (\cos x)^2 + (\cos x)^2 = 2(\cos x)^2 = f(x);\ D_{F'} = \mathbb{R} = D_f$

6. **a)** x-Achsenpunkte:
$f(x) = \sqrt{2} \cdot \sin x = 0;\ D_f =]-\frac{\pi}{12}; \frac{13\pi}{12}[;$
$\sin x = 0;\ x = k\pi;\ k \in \mathbb{Z};$
$S_1\ (0 \mid 0);\ S_2\ (\pi \mid 0)$
y-Achsenpunkt:
$f(0) = \sqrt{2} \cdot \sin 0 = 0;$
$T\ (0 \mid 0) = S_1$

b) $f'(x) = \sqrt{2} \cdot \cos x = 0;\ D_{f'} =]-\frac{\pi}{12}; \frac{13\pi}{12}[;$
$\cos x = 0;\ \ x = \frac{(2k+1)\pi}{2};\ \ k \in \mathbb{Z}$
Monotonietabelle:

x	$-\frac{\pi}{12} < x < \frac{\pi}{2}$	$x = \frac{\pi}{2}$	$\frac{\pi}{2} < x < \frac{13\pi}{12}$
$f'(x)$	$f'(x) > 0$	$f'(x) = 0$	$f'(x) < 0$
f	streng monoton zunehmend		streng monoton abnehmend
G_f	streng monoton steigend	Hochpunkt $H\left(\frac{\pi}{2} \mid \sqrt{2}\right)$	streng monoton fallend

c)



$f(x) = p(x);\ p(x) = -\frac{4}{\pi^2} \cdot x \cdot (x - \pi);$
$\sqrt{2} \cdot \sin x = -\frac{4}{\pi^2} \cdot x \cdot (x - \pi);$
$f'(x) = \sqrt{2} \cdot \cos x;\ p'(x) = -\frac{4}{\pi^2} \cdot (2x - \pi)$
Gemeinsame Punkte:
Man erkennt: $p(0) = 0$ und $f(0) = 0$; $A\ (0 \mid 0)$
$p(\pi) = 0$ und $f(\pi) = 0$; $B\ (\pi \mid 0)$
Bei Berührung von G_f und P müssen beide Graphen in A die gleiche Steigung besitzen und in B ebenfalls.
$f'(0) = \sqrt{2};\ p'(0) = \frac{4}{\pi} \neq \sqrt{2}$: A ist Schnittpunkt;
$f'(\pi) = -\sqrt{2};\ p'(\pi) = -\frac{4}{\pi} \neq -\sqrt{2}$: B ist Schnittpunkt.

d) $F'(x) = -a \cdot \sin x = f(x);\ D_{F'} =]-\frac{\pi}{12}; \frac{13\pi}{12}[;$
$-a \cdot \sin x = \sqrt{2} \cdot \sin x;$
$a = -\sqrt{2};$
$F(x) = -\sqrt{2} \cdot \cos x + b$

$F(0) = -\sqrt{2} \cdot \cos 0 + b = -\sqrt{2} + b = 0;\ b = \sqrt{2}$
$F\colon F(x) = -\sqrt{2} \cdot \cos x + \sqrt{2};\ D_F = D_f$