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	$f(x)$	x_0	$f'(x)$	$f(x_0)$	$f'(x_0)$
a)	$f(x) = e^x(x - 2)$	2	$f'(x) = e^x(x - 2) + e^x \cdot 1 =$ $= e^x(x - 1)$	0	$e^2 \approx 7{,}39$
b)	$f(x) = 2e^x - e^{2x}$	0	$f'(x) = 2e^x - 2e^{2x}$	1	0
c)	$f(x) = x^2e^{-x}$	1	$f'(x) = 2xe^{-x} - x^2e^{-x} =$ $= xe^{-x}(2 - x)$	$\frac{1}{e} \approx 0{,}37$	$\frac{1}{e}$
d)	$f(x) = e^{\ln(x^2)} = x^2$	e	$f'(x) = 2x$	$e^2 \approx 7{,}39$	$2e \approx 5{,}44$
e)	$f(x) = (1 + e\sqrt{x})^2$	9	$f'(x) = 2(1 + e\sqrt{x}) \cdot \frac{e}{2\sqrt{x}} =$ $= \frac{e}{\sqrt{x}}(1 + e\sqrt{x})$	$(1 + 3e)^2 \approx$ $\approx 83{,}81$	$\frac{e}{3}(1 + 3e) \approx 8{,}30$
f)	$f(x) = \ln(2x) - \ln(3x) =$ $= \ln\frac{2}{3}$	10	$f'(x) = 0$	$\ln\frac{2}{3} \approx 0{,}41$	0
g)	$f(x) = \ln\frac{1}{x} = -\ln x$	4	$f'(x) = -\frac{1}{x}$	$-\ln 4 \approx -1{,}39$	$-\frac{1}{4} = -0{,}25$
h)	$f(x) = e(e - x) = e^2 - ex$	e^2	$f'(x) = -e$	$e^2 - e^3 \approx -12{,}70$	$-e \approx -2{,}72$
i)	$f(x) = e^x(e^2 - 2)$	−1	$f'(x) = e^x(e^2 - 2)$	$\frac{e^2 - 2}{e} \approx 1{,}98$	$\frac{e^2 - 2}{e}$
j)	$f(x) = (1 - \sqrt{6x})^3$	6	$f'(x) = 3(1 - \sqrt{6x})^2 \cdot \frac{-6}{2\sqrt{6x}} =$ $= -\frac{9(1 - \sqrt{6x})^2}{\sqrt{6x}}$	−125	$-\frac{9 \cdot (-5)^2}{6} = -37{,}5$
k)	$f(x) = 1 + \ln\sqrt{ex} =$ $= 1 + \frac{1}{2}\ln e + \frac{1}{2}\ln x$	e	$f'(x) = \frac{1}{2} \cdot \frac{1}{x} = \frac{1}{2x}$	$1 + \ln e \approx 2$	$\frac{1}{2e} \approx 0{,}18$
l)	$f(x) = \frac{e^x - e^{-x}}{2}$	1	$f'(x) = \frac{e^x + e^{-x}}{2}$	$\frac{1}{2}\left(e - \frac{1}{e}\right) \approx 1{,}18$	$\frac{1}{2}\left(e + \frac{1}{e}\right) \approx 1{,}54$
m)	$f(x) = (\ln x)^2$	2	$f(x) = \frac{2\ln x}{x}$	$(\ln 2)^2 \approx 0{,}48$	$\ln 2 \approx 0{,}69$
n)	$f(x) = \ln(x^2)$	−2	$f'(x) = \frac{2x}{x^2} = \frac{2}{x}$	$\ln 4 \approx 1{,}39$	−1

1. Bilden Sie jeweils $f'(x)$ und geben Sie $f(x_0)$ an ($D_f = D_{f\,max}$).

	$f(x)$	x_0	$f(x_0)$	$f'(x)$
a)	$f(x) = x\sqrt{x} = x^{\frac{3}{2}}$	1	1	$f'(x) = \frac{3}{2}x^{\frac{1}{2}} - \frac{3}{2}\sqrt{x}$
b)	$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$	8	2	$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3 \cdot \sqrt[3]{x^2}}$
c)	$f(x) = \cos\sqrt{x}$	π^2	-1	$f'(x) = (-\sin\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} = -\frac{1}{2\sqrt{x}} \sin\sqrt{x}$
d)	$f(x) = \sin\frac{1}{\sqrt{x}}$	$\frac{1}{\pi^2}$	0	$f'(x) = \left(\cos\frac{1}{\sqrt{x}}\right) \cdot \left(-\frac{1}{2}\right)x^{-\frac{3}{2}} = -\frac{1}{2\sqrt{x^3}} \cos\frac{1}{\sqrt{x}}$
e)	$f(x) = \sqrt[3]{x^2 + 26}$	1	3	$f'(x) = \frac{1}{3} \cdot \frac{2x}{\sqrt[3]{(x^2+26)^2}} = \frac{2x}{3 \cdot \sqrt[3]{(x^2+26)^2}}$
f)	$f(x) = \sqrt{4 - (2-x)^2} =$ $\sqrt{4 - 4 + 4x - x^2} = \sqrt{4x - x^2}$	2	2	$f'(x) = \frac{4 - 2x}{2\sqrt{4x - x^2}} = \frac{2 - x}{\sqrt{4x - x^2}}$
g)	$f(x) = x^n \cdot x^{1+n} = x^{2n+1}$; $n \in \mathbb{Z}$	1	1	$f'(x) = (2n + 1)x^{2n}$
h)	$f(x) = (2 - \sqrt{x})^{1,5}$	1	1	$f'(x) = 1{,}5 \cdot (2 - \sqrt{x})^{\frac{1}{2}}\left(-\frac{1}{2\sqrt{x}}\right) = -\frac{3\sqrt{2 - \sqrt{x}}}{4\sqrt{x}}$
i)	$f(x) = \{[1 - [\cos(2x)]^2\}^{0,5} =$ $= \lvert\sin(2x)\rvert = \sin(2x)$, da $\sin(2x) > 0$	0	0	$f'(x) = 2\cos(2x)$
j)	$f(x) = [8 - (5 - 2x)]^3 = (3 + 2x)^3$	2	343	$f'(x) = 3(3 + 2x)^2 \cdot 2 = 6(3 + 2x)^2$
k)	$f(x) = \sqrt{a^2x^2} + a^3 =$ $= \lvert ax\rvert + a^3 = ax + a^3$, da $a > 0$ und $x > 0$	$\sqrt{a}$	$a\sqrt{a} + a^3$	$f'(x) = a$
l)	$f(x) = \sqrt{a^2x^2 + a^3} = a\sqrt{x^2 + a}$	1	$a\sqrt{1 + a}$	$f'(x) = a \cdot \frac{1}{2} \cdot \frac{2x}{\sqrt{x^2 + a}} = \frac{ax}{\sqrt{x^2 + a}}$
m)	$f(x) = \frac{4}{\sqrt{x^2 + 1}} = 4(x^2 + 1)^{-0,5}$	0	4	$f'(x) = 4 \cdot \left(-\frac{1}{2}\right) \cdot (x^2 + 1)^{-1,5} \cdot 2x = -\frac{4x}{\sqrt{(x^2+1)^3}}$

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	Funktionsterm f(x)	x_0	f'(x)	$f'(x_0)$
a)	$f(x) = \frac{2}{x}$	$\frac{1}{2}$	$f'(x) = -\frac{2}{x^2}$	$f'(0{,}5) = -8$
b)	$f(x) = \frac{7}{x^2}$	−2	$f'(x) = -\frac{14}{x^3}$	$f'(-2) = \frac{14}{8} = \frac{7}{4}$
c)	$f(x) = \left(2x - \frac{2}{x}\right)^2 = 4x^2 - 8 + \frac{4}{x^2}$	−1	$f'(x) = 8x - \frac{8}{x^3}$	$f'(-1) = -8 + 8 = 0$
d)	$f(x) = \frac{x-1}{x+1}$	2	$f'(x) = \frac{(x+1) \cdot 1 - (x-1) \cdot 1}{(x+1)^2} = \frac{2}{(x+1)^2}$	$f'(2) = \frac{2}{9}$
e)	$f(x) = \frac{3}{x^2 + x}$	1	$f'(x) = -\frac{3(2x+1)}{(x^2+x)^2}$	$f'(1) = -\frac{9}{4}$
f)	$f(x) = \frac{4}{4x+1}$	$\frac{1}{4}$	$f'(x) = \frac{-4 \cdot 4}{(4x+1)^2} = -\frac{16}{(4x+1)^2}$	$f'(0{,}25) = -4$
g)	$f(x) = \frac{1}{32}x^4 + \frac{32}{x^4}$	2	$f'(x) = \frac{4}{32}x^3 - \frac{32 \cdot 4}{x^5} = \frac{x^3}{8} - \frac{128}{x^5}$	$f'(2) = 1 - 4 = -3$
h)	$f(x) = 2x + 2 + \frac{1}{x-1}$	0	$f'(x) = 2 - \frac{1}{(x-1)^2}$	$f'(0) = 2 - 1 = 1$
i)	$f(x) = \left(x^2 - \frac{1}{x^2}\right)^2 = x^4 - 2 + \frac{1}{x^4}$	−1	$f'(x) = 4x^3 - \frac{4}{x^5}$	$f'(-1) = -4 + 4 = 0$
j)	$f(x) = \left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right) = x^2 - \frac{1}{x^2}$	$\frac{1}{2}$	$f'(x) = 2x + \frac{2}{x^3}$	$f'(0{,}5) = 1 + 16 = 17$
k)	$f(x) = \frac{x}{x^2+1}$	0	$f'(x) = \frac{(x^2+1) \cdot 1 - x \cdot 2x}{(x^2+1)^2} = \frac{x^2 + 1 - 2x^2}{(x^2+1)^2} = \frac{1 - x^2}{(x^2+1)^2}$	$f'(0) = 1$
l)	$f(x) = \frac{x^2+1}{x} = x + \frac{1}{x}$	0,5	$f'(x) = 1 - \frac{1}{x^2}$	$f'(0{,}5) = 1 - 4 = -3$